

Constrained Single-Point Aerodynamic Shape Optimization of the DPW-W1 wing through Evolutionary Programming and Support Vector Machines

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Summary

The application of surrogate-based methods to the constrained optimization of aerodynamic shapes is nowadays a very active research field due to the potential of these methods to reduce the number of actual computational fluid dynamics simulation runs, and therefore drastically speed-up the design process. However, their feasibility when handling a large number of design parameters, which in fact is the case in industrial configurations, remains unclear and needs further efforts, as demonstrated by recent research on design space reduction techniques and adaptive sampling strategies. This paper presents the results of applying surrogate-based optimization to the three-dimensional, constrained aerodynamic shape design of the DPW-W1 wing, involving both inviscid and viscous transonic flow. The wing geometry is parameterized by a control box with 36 design variables and the applied approach is based on the use of Support Vector Machines (SVMs) as the surrogate model for estimating the objective function, in combination with an Evolutionary Algorithm (EA) and an adaptive sampling technique focused on optimization, called the Intelligent Estimation Search with Sequential Learning (IES-SL).

Keywords: *Aerodynamic shape design, surrogate modeling, evolutionary programming, support vector machines, constrained optimization, transonic wing design*

1 Introduction and previous works

1.1. Introduction

In the last few years, there has been an increasing interest in the topic of Surrogate-based Optimization (SBO) methods for aerodynamic shape design. This is due to the promising potential of these methods to speed-up the design process by the use of a “low cost” objective function evaluation to reduce the required number of expensive computational fluid dynamics (CFD) simulations. However, the application of these SBO methods for industrial configurations still requires facing several challenges, such as the so-called “curse of dimensionality”, the ability of surrogates when handling a high number of design parameters, efficient constraints handling, adequate exploration and exploitation of the design space, etc.

1.2. Recent research efforts in SBO for aerodynamic shape design

A physics-based surrogate model was recently applied in [1] to the drag minimization of a NACA0012 airfoil in inviscid transonic flow and a RAE2822 airfoil in viscous transonic flow, both using the PARSEC parameterization with up to ten design parameters. The drag minimization problem was also addressed by SBO in [2] for the NLF0416 airfoil, parameterized with ten design parameters.

Moreover, a combination of a genetic algorithm (GA) and an artificial neural network (ANN) was applied in [3] to the

shape optimization of an airfoil, parameterized by a modified PARSEC parameterization involving ten design variables. In [4] a surrogate based on Proper Orthogonal Decomposition (POD) was applied to the aerodynamic shape optimization of an airfoil geometry parameterized by sixteen design variables defined with Class Shape Transformation method (CST). In summary, the ability of SBO methods to manage a high number of design parameters still remains an open challenge and have been studied by several authors in the last few years, as well as the strategies for efficient infill sampling criteria with constraint handling [4, 5].

Finally, the authors also presented recent works on this topic [6, 7]. This paper is an extension of previous research, here considering the constrained single-point aerodynamic optimization of the DPW-W1 wing for both inviscid and viscous transonic flow.

1.3. GARTEUR AD/AG52

This work is part of a GARTEUR (Group for Aeronautical Research and Technology in Europe, www.garteur.org) Action Group that was established to explore these SBO approaches. The main objective of the AG [8] is, by means of a European collaborative research, to make a deep evaluation and assessment of surrogate-based global optimization methods for aerodynamic shape design.

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2 Definition of the optimization problem

2.1. Baseline geometry: DPW-W1 wing

The public domain transonic DPW-W1 wing (a test case of the Third AIAA Drag Prediction Workshop) was used [9, 10]. Reference quantities for this wing are displayed in the following table:

Sref (wing ref. area)	290322 mm ²
Cref (wing ref. chord)	197.55 mm
Xref	154.24 mm (relative to the wing root leading edge)
b/2 (semi span)	762 mm
AR (aspect ratio, AR=b ² /Sref)	8.0

Table 1: DPW reference quantities

The initial geometry (in IGES format) was downloaded from [9]. A set of grids are also available in the website of the 3rd AIAA Workshop on Drag Prediction.

2.2. Parameterization

The DPW geometry is parameterized by a 3D control box (displayed in Figure 1) with 5 control points in direction u, 10 in direction v and 5 in direction w. The parametric u direction corresponds to the y axis, the v direction to the x axis, and the w direction to the z axis.

The design variables are the vertical displacement of those control points set up on the aerodynamic surface. The wing is split in three profile sections and the transition between sections is linear. Each section has 6 active control points for the upper side and other 6 for the lower side, which are independent (the movement of a control point at the upper side does not modify the lower side and *vice versa*), with a total of 36 design parameters for the whole wing. Authors have previously applied this parameterization technique to other local and global optimization problems [11]. During the optimization performed in this paper, the wing platform will be kept fixed, as well the angle of attack and the torsion.

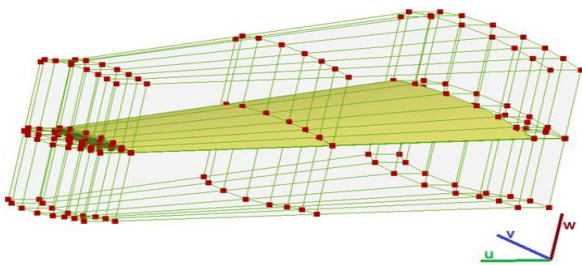


Figure 1: DPW wing parameterization

2.3. Aerodynamic constraints

The following aerodynamic constraints are considered:

- 1) Prescribed constant lift coefficient ($C_L = C_L^0$)
- 2) Minimum pitching moment: $C_M > = C_M^0$
- 3) Drag penalty: If constraint in minimum pitching moment is not satisfied, the penalty will be 1 drag count per 0.01 increment in C_M .

2.4. Geometric constraints

Each design variable will be constrained by its minimum and maximum values that will be chosen as the + or - 20% of their original value. Apart from this, other constraints have been defined, according to [10]:

- 1) Airfoils' maximum thickness constraints:

$$(t/c)_{section} \geq (t/c)_{section}^0$$

where the right term is the maximum thickness for the original wing sections, root, mid-span and tip, which has the value of 13.5%.

- 2) Beam constraints

First, two locations (x/c) are fixed to represent the beam constraints:

$$(x/c)_{root,1} = (x/c)_{mid-span,1} = (x/c)_{tip,1} = 0.20$$

$$(x/c)_{root,2} = (x/c)_{mid-span,2} = (x/c)_{tip,2} = 0.75$$

The constraint here is that the thickness value of the optimized wing sections at these locations should be greater or equal than the thickness of the original ones. It is defined with the expressions:

$$(t/c)_{root,1} \geq 12\%, (t/c)_{mid-span,1} \geq 12\%, (t/c)_{tip,1} \geq 12\%$$

$$(t/c)_{root,2} \geq 5.9\%, (t/c)_{mid-span,2} \geq 5.9\%, (t/c)_{tip,2} \geq 5.9\%$$

2.5. Design point and objective function

This paper addresses a single-point optimization of the DPW-W1 wing, for both inviscous and viscous transonic flow. Multi-point optimization will be also considered as a future work within the GARTEUR AG52 group [8]. The flow conditions are: Mach number 0.8, an angle of attack of zero degrees and a Reynolds number of $5 \cdot 10^6$. The design goal is to achieve a geometry with the minimum drag, while maintaining the specified aerodynamic constraints. Aerodynamic constraints are implemented as penalties in the objective function. The pseudo-code implementation is:

```
lift_penalty=1-(Cl/C10);
if (lift_penalty<0) lift_penalty=0;
cm_penalty = (Cm0-Cm)*0.0001/0.01;
if (cm_penalty < 0) cm_penalty = 0;
objective_function=((Cd+cm_penalty)/Cd0 )+5*lift_penalty;
```

2.6. Computational grids

The following unstructured grids were used:

	#points	#surface points	#elements	#surface elements
DPW-EULER	427k	135k	2112k	276k
DPW RANS*	3770k	152k	9335k	310k

Table 2: Computational grids

*The DPW RANS grid was downloaded directly from the 3rd Workshop on Drag Prediction web page.

3 Description of the applied approach

3.1. Adaptive sampling focused on optimization

The Intelligent Estimation Search with Sequential Learning (IES-SL) is an algorithm designed to implement an adaptive sampling directly focused on the optimization search. From this point of view, the key feature of this novel approach is to use the surrogate model to estimate the location of the optimum in the real function. To do this, an optimization search is applied over the surrogate, obtaining an estimated value of the real minimum position (an ‘‘intelligent guess’’). Each of the estimations of the optimum location gives us a new sampling point (it means a new geometry that is also analyzed using the high fidelity CFD solver). Within a try-and-error cycle, the surrogate proposes a new design which is again evaluated by the CFD solver and then, in a sequential learning, the surrogate model is enriched with the associated cost function.

3.2. Support Vector Regression algorithm as surrogate model

SVMs represent appealing algorithms for a large variety of regression problems due to they do not only take into account the error approximation to the data, but also the generalization of the model, namely, their capability to improve the prediction of the model when new data are evaluated. This kind of methods can be considered a specific type of ANNs, and are commonly trained by means of a deterministic method known as Sequential Minimal Optimization (SMO) which provides a significant computational complexity reduction. The used SVM method for regression consists of, given a set of training vectors $C = \{(x_i, y_i), i = 1, \dots, l\}$, training a model of the form $y(x) = f(x) + b = w^T \phi(x) + b$, to minimize a general risk function of the form

$$R[f] = \frac{1}{2} \|w\|^2 + \frac{1}{2} C \sum_{i=1}^l L(y_i, f(x)) \quad (1)$$

where w controls the smoothness of the model, $\phi(x)$ is a function of projection of the input space to the feature space, b is a parameter of bias, x_i is a feature vector of the input space with dimension N , y_i is the output value to be estimated and $L(y_i, f(x))$ is the loss function selected. In this paper, the L1-SVR (L1 support vector regression) is used, characterized by an ε -insensitive loss function

$$L(y_i, f(x)) = |y_i - f(x_i)|_{\varepsilon} \quad (2)$$

In order to train this model, it is necessary to solve the following optimization problem

$$\min \left(\frac{1}{2} \|w\|^2 + \frac{1}{2} C \sum_{i=1}^l (\xi_i + \xi_i^*) \right) \quad (3)$$

subject to:

$$\begin{aligned} y_i - w^T \phi(x_i) - b &\leq \varepsilon + \xi_i, i = 1, \dots, l \\ -y_i + w^T \phi(x_i) + b &\leq \varepsilon + \xi_i^*, i = 1, \dots, l \\ \xi_i, \xi_i^* &\geq 0, i = 1, \dots, l \end{aligned} \quad (4)$$

The SVM can use different kernels to face non-linear problems.

On this case, a radial basis function has been used as a kernel function. This training procedure must be combined with the search of three parameters (C , ε , and γ , named hyperparameters) on which the final model depends. The influence of the three parameters on the SVM model can be seen on equations (3) where C defines the optimization problem and in equation (4) where ε represents the constraints for the optimization problems. Finally, the radial basis kernel depends on the value of γ . To obtain the best SVM performance, a search of the most suitable combination of these three parameters must be carried on, usually by using cross validation techniques over the training set. To reduce the computational time of this process, different methods have been proposed in the literature to reduce the search space related to these parameters. In this case, it has been applied the one developed in [12], which has proven to require pretty short search times.

3.3. Evolutionary programming

The EA implemented for this work has the following characteristics: the selection operator is applied by replacing a portion of the current generation by new individuals generated from parents. It is considered the replacement of the individuals in the population with fitness value under the population’s mean fitness. A multipoint crossover which selects the value of one of the parents with probability 0.5 is applied. Regarding the mutation operator, the values of each new individual are mutated with probability $1/N_p$, where N_p is the number of parameters to be optimized. A mutation parameter can be tuned in order to allow a more global or local search over a certain design variable. More detailed information about the implemented algorithm can be found in [7].

3.4. Handling constrains within the optimization process

In the context of evolutionary optimization, constraints can be handled by adding penalties to the objective function. These penalties can be imposed in a ‘soft’ or ‘hard’ manner. Soft penalties increment the unconstrained objective function proportionally with information about how far from the constraint is a certain solution. This kind of penalties allows the system to work with non-feasible but interesting solutions, improving the search space and finally obtaining feasible solutions. Hard penalties imply that the restriction must be fulfilled at any time of the optimization process. In this case, the solutions are strongly penalized and therefore removed from the search process. The constrained objective function is represented by the following expression:

$$f_{obj} = f_{obj}^* + soft_penalt + hard_penalt \quad (5)$$

where f_{obj}^* is the unconstrained objective function.

In this paper, the constraints within the SBO process are handled in the following way. First, the simulation system M allows computing the aerodynamic characteristics (i.e. C_D , C_L , C_M) of a geometry defined by a set of parameters P .

$$[C_D, C_L, C_M] = M(P) \quad (6)$$

The objective function to be minimized, including the mentioned constraints, can be described as a combination of the output factors from the simulation system (aerodynamic characteristics), and other factors associated with the model (i.e. geometric characteristics). This function can be represented as:

$$fobj = f(C_D, C_L, C_M, P) \quad (7)$$

where the geometric restrictions are calculated from the set of model parameters P and the aerodynamic constraints are directly computed from the aerodynamic characteristics. Since the application of the simulator system to obtain the aerodynamic values is very expensive, the surrogate model is added to the system to reduce the computational cost of the optimization process. There are three different approaches to use the surrogate model to speed up the constrained objective function computation.

The first option is to generate a surrogate that directly models the objective function. This is the simplest and more direct method to apply the surrogate. The individual evaluation on this case is carried on by the surrogate model (i.e. the SVM).

$$fobj = SVM(P) \approx f(C_D, C_L, C_M, P) \quad (8)$$

The second option is to extract from the objective function the restrictions that are independent from the simulator output. In this case, the geometric constraints can be calculated independently:

$$\begin{aligned} fobj &= f(C_D, C_L, C_M, P) = \\ &= f1(C_D, C_L, C_M) + f2(P) \\ SVM(P) &\approx f1(C_D, C_L, C_M) \end{aligned} \quad (9)$$

$$fobj = SVM(P) + f2(P) \approx f(C_D, C_L, C_M, P)$$

This division reduces the complexity of the surrogate model, because it does not have to model the geometric information (only aerodynamic features). On the other hand, the system must perform an additional computation since in each evaluation the system must compute both the SVM output and the value of $f2$.

The third option is another step to simplify the surrogate model. The penalties in the objective function can add additional complexity to the function like discontinuities around the restriction boundaries. This can reduce the quality that a surrogate can achieve with a fixed number of data points. To avoid this effect, a multi-surrogate model can be implemented, and this is the approach considered in this paper. Each simulator output (i.e. C_D , C_L , C_M), is modeled by a different SVM (SVM_Cd, SVM_Cl, SVM_Cm), and then applied to the $f1$ function that contains the aerodynamic restrictions.

$$\begin{aligned} fobj &= f1(SVM_Cd(P), SVM_Cl(P), SVM_Cm(P)) \\ &+ f2(P) \approx f(C_D, C_L, C_M, P) \end{aligned} \quad (10)$$

In this way, the penalties associated to $f1$, and their corresponding complexity, are added after building the surrogate, allowing achieving simpler SVM models, with higher quality and accuracy. On the other hand, the global system is more complex, since now it is necessary to train now three different surrogates.

4 Numerical results

The proposed approach is applied to the constrained single point optimization of the DPW-W1 wing in both inviscid and viscous transonic flow conditions. In order to handle the geometric constraints previously defined, the parameterization is prepared by locating certain control points in specific locations, as displayed in Figure 2.

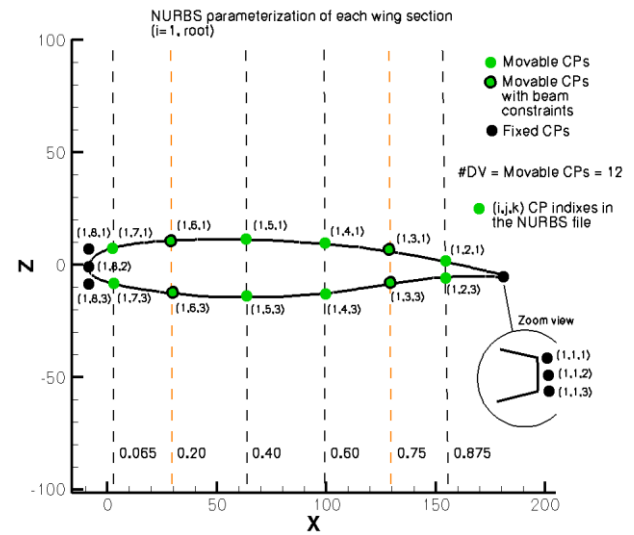


Figure 2: Geometric constraints handling through the selected control box parameterization (wing section)

4.1. Inviscid transonic flow

The following table shows the objective function of the original and optimized geometries. The results show that the objective function has been improved by 23% (after 192 iterations), while both aerodynamic and geometric the constraints have been satisfied.

	fobj	Cd	Cl	Cm
DPW-W1	1	0.0307	0.5984	-0.02867
Optimized	0.77	0.0236	0.5981	-0.02653

Table 3: Objective function and aerodynamic coefficients of baseline and optimized geometries

Figures 3, 4 and 5 show the Mach number distribution, shapes and C_p comparison between the baseline geometry and the optimized shape.

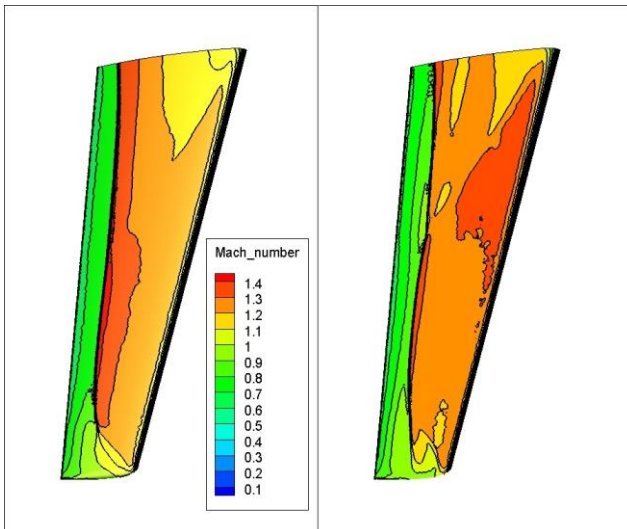


Figure 3: Mach number distribution on the original (left) and optimized (right) geometries

The computational time for the whole optimization of the Euler case using 8 processors on a Linux x86_64 computational cluster was about 40 hours.

4.2. Viscous transonic flow

The following table shows the objective function of the original and optimized geometries. The results show that the objective function has been improved by 5%, while the constraints have been satisfied. In the full paper, the complete final results will be included, together with a grid sensitivity analysis in order to ensure that the optimization achieved is not due to grid issues.

	fobj	Cd	Cl	Cm
DPW-W1	1	0.0257	0.3636	-0.0687
Optimized	0.95	0.0245	0.3658	-0.0684

Table 4: Objective function and aerodynamic coefficients of baseline and optimized geometries

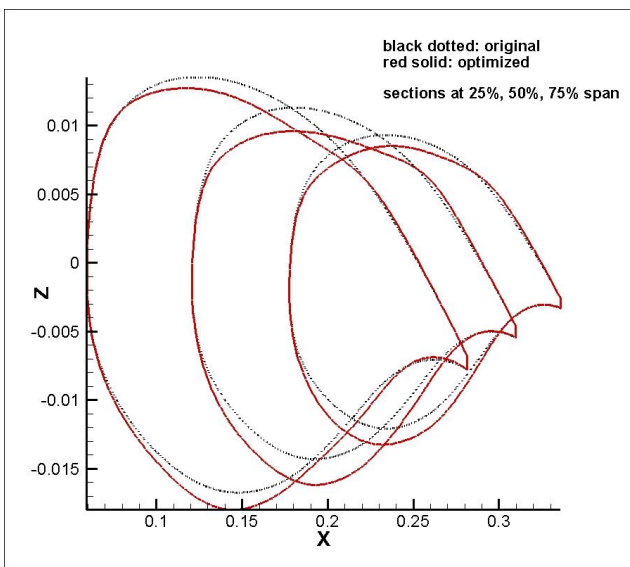


Figure 4: Original vs. optimized geometries

Figures 6, 7 and 8 show the shapes and Cp comparison between the baseline geometry and the optimized shape, which was obtained in the iteration number 175.

The computational time for 175 iterations of the RANS case using 36 processors on a Linux x86_64 computational cluster was about 170 hours (7 days).

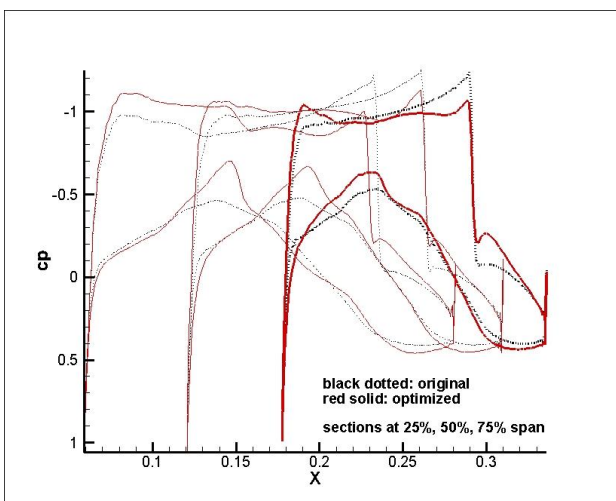


Figure 5: Cp plots along wing span

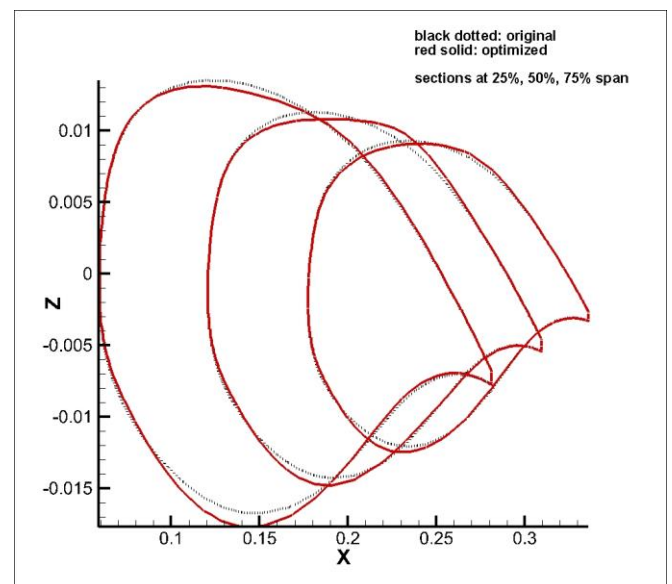


Figure 6: Original vs. optimized geometries (preliminary results, optimization process not finished)

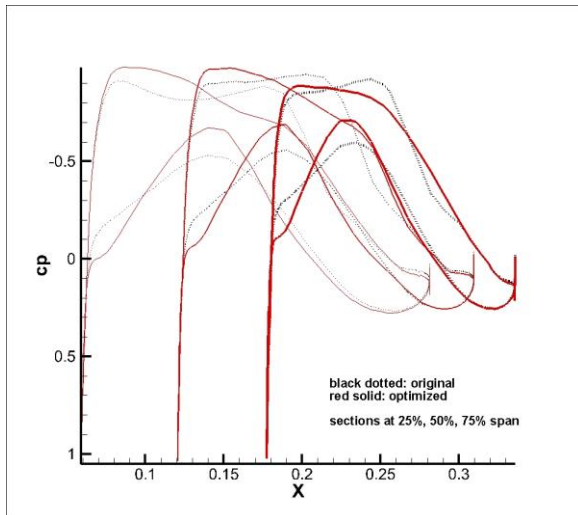


Figure 7: C_p plots along wing span (preliminary results, optimization process not finished)

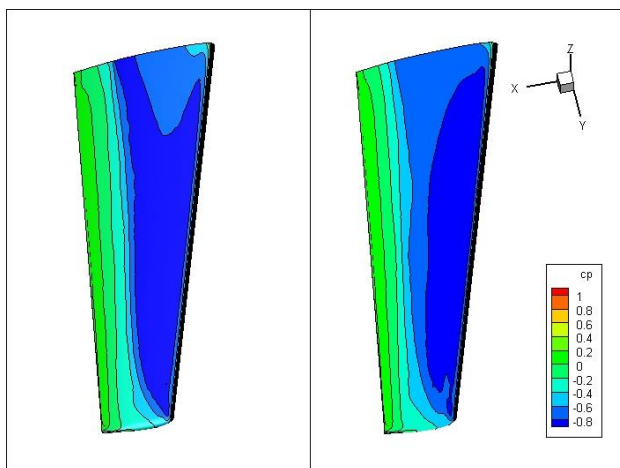


Figure 8: C_p distribution on the original (left) and optimized (right) geometries

5 Conclusions

This paper presented the application of a global optimization strategy using the Intelligent Estimation Search with Sequential Learning (IES-SL) and the hybridization of EA and SVMr to the single-point constrained optimization of a three dimensional DPW wing in both inviscid and viscous transonic flow conditions, showing first promising results.

Future work will address the multi-point constrained optimization, for comparison with the results obtained by Epstein and Jameson in [10]. This extension will be performed within the GARTEUR AG52 group. In addition, research work on the parameterization sensitivity to the SBO process is also being performed.

6 Acknowledgements

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7 References

- [1] Leifsson L, Koziel S., and Tesfahunegn Y. Aerodynamic Design Optimization: Physics-based Surrogate Approaches for Airfoil and Wing Design. AIAA SciTech 2014. AIAA 2014-0572
- [2] Koziel S. and Leifsson L. Multi-Level Surrogate-Based Airfoil Shape Optimization. 51st AIAA Aerospace Sciences Meet-ing including the New Horizons Forum and Aerospace Ex-position 07 - 10 January 2013, Grapevine (Dallas/Ft. Worth Region), Texas AIAA2013-0778.
- [3] Jahangirian, A., and Shahrokhi, A. 2011. Aerodynamic shape optimization using efficient evolutionary algorithms and unstructured CFD solver. *Computers & Fluids*, 46, 270–276.
- [4] Iuliano E. and Quagliarella D. Aerodynamic Shape Optimization via non-intrusive POD-based Surrogate Modelling. 2013 IEEE Congress on Evolutionary Computation June 20-23, Cancún, México.
- [5] Parr J., Holden C., Forrester A., Keane A. Review of Efficient Surrogate Infill Sampling Criteria with Constraint Handling. 2nd International Conference on Engineering Optimization Sept., 2010, Lisbon, Portugal.
- [6] Andres E., Salcedo-Sanz S., Monge F., Perez-Bellido A. M. 2012. Efficient aerodynamic design through evolutionary programming and support vector regression algorithms. *Expert Systems with Applications* 39 (2012) 10700–10708. Sept. 2012.
- [7] Andres E., Carro-Calvo L, Salcedo-Sanz S. 2014. Fast aerodynamic coefficients prediction using SVMs for global shape optimization. ECCOMAS CFD, Barcelona, June 2014.
- [8] Gar-teur AD/AG52 members 2013. Partial Report on Surrogate-based Global Optimization Methods in Aerodynamic Design. <https://ag52.blogspot.com>
- [9] Web site of the 3rd AIAA Workshop on Drag Prediction: <http://aaac.larc.nasa.gov/tsab/cfdlarc/aiaa-dpw/Workshop3/workshop3.html>
- [10] Epstein B., A. Jameson, S. Peigin, D. Roman, N. Harrison, J. Vassberg. Comparative Study of 3D Wing Drag Minimization by Different Optimization Techniques, 46th AIAA Aerospace Sciences Meeting and Exhibit. Jan. 2008, Reno, Nevada. AIAA2008-326.
- [11] Martin M. J. , Andres E., Valero E., Lozano C. Gradients Calculation for Arbitrary Parameterizations via Volumetric NURBS: The Control Box Approach. EUCASS conference 2013.
- [12] Ortiz-García E. G., S. Salcedo Sanz, Á M. Pérez-Bellido, J. A. Portilla-Figueras. Improving the training time of support vector regression algorithms through novel hyper-parameters search space reductions. *Neurocomputing*, 2009.