

## GARTEUR AD/AG-52: SURROGATE-BASED GLOBAL OPTIMIZATION METHODS IN PRELIMINARY AERODYNAMIC DESIGN

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### Summary

This work presents a summary of the results obtained during the activities developed within the GARTEUR AD/AG-52 group. GARTEUR stands for “Group for Aeronautical Research and Technology in Europe” and is a multinational organization that performs high quality, collaborative, precompetitive research in the field of aeronautics to improve technological competence of the European Aerospace Industry. The aim of the AG52 group was to make an evaluation and assessment of surrogate-based global optimization methods for aerodynamic shape design of aeronautical configurations. The structure of the paper is as follows: Section 1 will introduce the state-of-the-art in surrogate-based optimization for aerodynamic design and section 2 will detail the test cases selected in the AG52 group. Optimization results will be then presented in section 3, and conclusions will be provided in the last section.

**Keywords:** *aerodynamic shape design, evolutionary optimization, computational fluid dynamics, surrogate-based optimization, surrogate modelling.*

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## 1 Introduction

The AD/AG 52 has been established to explore and unveil the potential of surrogate-based techniques in aerodynamic shape optimization. Any designer has experienced the burden of intensive numerical optimization involving CFD or analogous expensive black-box simulations. Typically, the computational load is easily tolerated when dealing with two-dimensional airfoil shape design. However, the order of magnitude of both the number of simulations required and the CPU time for single evaluation grows significantly with increasing dimensionality (e.g. from two-dimensional to three-dimensional cases) and geometric complexity (e.g., wing-fuselage configuration, high-lift cases, wing-pylon-engine installation). Surrogate models are able to complement, not to replace, the “true” function evaluation by providing a fast and adaptive response during screening parametric analyses and numerical optimization. Building and querying a surrogate is a way to potentially acquire new information about the problem under analysis, not to directly solve it: indeed, any surrogate, even the most sophisticated ones, have to address prediction error and minimize it in order to be accurate. This does not hamper the usefulness of the approach as, even in presence of errors away from the sampled data, function trends and optimization directions can be derived to enrich the process.

The main objective of this Action Group was to make a comprehensive evaluation and assessment of surrogate-based global optimization methods for aerodynamic shape optimization.

The work structure for this AG is application-driven, and it was composed of two tasks. First, in Task 1, two common test cases were proposed and addressed by all partners using different methods. The aim was to make an exhaustive comparison of promising methods and to quantify their performance in terms of accuracy and CPU cost. Then, in Task 2, more industry-relevant test cases were provided, and the consortium used the knowledge acquired in task 1, to solve such test cases.

### 1.1. State-of-art

Global search methods are traditionally based on stochastic optimization techniques; most of them are population-based whereas there are few individual-based algorithms. The most commonly used population-based methods are the Evolutionary Algorithms (EAs; including Genetic Algorithms-GAs and Evolution Strategies-ES). However, other alternatives exist, such as Particle Swarm Optimization (PSO) [1], Bacterial Foraging Optimization (BFO) [2] and Differential Evolution (DE) [3]. Evolutionary Algorithms (EAs) [4] are successful single and multi-objective constrained optimization methods that can handle any kind of objective function and may accommodate any evaluation software as a black-box tool. Due to the high and expensive number of required calls, EAs assisted by surrogate

evaluation models (metamodels) have been devised and, depending on the training method, they can be classified as off-line trained metamodels [5, 6, 7], or on-line trained metamodels [8, 9, 10, 11].

There are different many types of surrogate modelling approaches, including Polynomial Regression (PR), Multivariate Adaptive Regression Splines (MARS), Gaussian Processes, Kriging (KG), Co-kriging [12], Artificial Neural Networks (ANN) [13, 14], Radial Basis Functions (RBF) [15], Proper Orthogonal Decomposition (POD) methods [16] and Support Vector Machines (SVM) [17, 18]. A reference for recent advances in surrogate based optimization techniques can be found in [19], and comparison of surrogate models for turbomachinery design in [20]. Also, surrogate modelling has been already applied for the design optimization of composite aircraft fuselage panels [21]. In addition, the use of Kriging surrogate modelling in combination with Evolutionary Algorithms has been recently applied for the design of hypersonic vehicles [22]. Furthermore, the use of Support Vector Regression algorithms (SVMr) as metamodels has been applied to a large variety of regression problems, frequently combined with evolutionary computation algorithms [23-26].

Current research focuses on the improvement of metamodels (by using Artificial Neural Networks, Gaussian models, etc. or proposing metamodels variants [27] based on not only the responses but also the gradient of responses, Kriging) and/or different metamodel implementation schemes within the Metamodel-Assisted Evolutionary algorithm (MAEA) [28]. Particular attention is required in multi-objective optimization problems, where a Pareto front of non-dominated solutions is sought and the evolving individuals are dispersed in the design space, or when asynchronous MAEAs [29, 30] are devised by overcoming the notion of generation and the corresponding synchronization barrier.

With respect to the combination of global and local search methods within the design optimization process, the so-called hierarchical approach has been proposed in the literature (for instance, stochastic methods for the exhaustive search of the design space along with gradient-based methods for the refinement of promising solutions) [31, 32]. Metamodel-assisted memetic algorithms [30] are also hybrid schemes that combine the use of global and local optimization methods [33-35].

## 2 Definition of common test cases & methods

Two test cases were selected to assess and compare methods: the RAE2822 airfoil and the Drag Prediction Workshop (DPW) W1 wing. The first is two-dimensional and it has been widely studied in the aerospace community over the last few decades; a large amount of both experimental and computational data exists, together with optimization results

generated by a variety of methodologies. Transonic viscous flow conditions were considered for this test case.

The second test case was taken as the DPW-W1 wing which has been proposed during the 3rd AIAA Drag Prediction Workshop [37]: it is a quite simple wing geometry that can be easily handled in an optimization context. Again, experimental and computational data are available for comparison. Transonic viscous flow conditions were also considered.

### 2.1. RAE2822 airfoil

The RAE 2822 airfoil [36] had been selected as the initial geometry for aerodynamic optimizations. The airfoil contour shape is shown in Figure 1 and Table 1 summarizes its geometrical characteristics.

Fig. 1. RAE2822 baseline geometry



Table 1. Baseline airfoil features

Chord [m]	0.61
Maximum thickness-to-chord ratio	0.121 @ $x/c = 0.38$
Maximum camber-to-chord ratio	0.0126 @ $x/c = 0.76$
Leading edge radius [m]	0.00827
Airfoil area [m <sup>2</sup> ]	0.0776
Trailing edge angle	9°

The flow conditions and constraints of different design points were the inputs for the optimization process. These flow conditions included prescribed angle of attack (AoA), Mach number, Reynolds number as follows:

- DP1 (Case 9):  $M = 0.734$ ,  $Re = 6.5 \cdot 10^6$ ,  $AoA = 2.65^\circ$
- DP2 (Case 10):  $M = 0.754$ ,  $Re = 6.2 \cdot 10^6$ ,  $AoA = 2.65^\circ$

The objective function defined was to maximize lift over drag ratio at both the design points, while maintaining some specified constraints.

The aerodynamic constraints and penalties considered were:

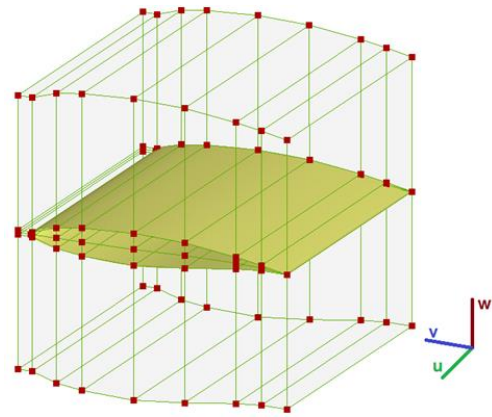
- Prescribed minimum lift coefficient:  
 $C_l^0|_k: C_l|_k \geq C_l^0|_k$
- Prescribed minimum pitching moment coefficient  
 $C_m^0|_k: C_m|_k \geq C_m^0|_k$ , where  $C_l^0|_k$  and  $C_m^0|_k$  are the lift and pitching moment coefficients, respectively, of the initial geometry, for the design point  $k$ .
- Drag penalty: if the constraint on minimum pitching moment is not satisfied, the penalty will be 1 drag count per 0.01 in  $\Delta C_m$ .

while the geometric constraints were:

- Prescribed maximum thickness ratio:  
 $(t/c)_{\max}: \max(t/c) = (t/c)_{\max}$
- Prescribed minimum thickness ratio  $(t/c)_{\min}^{80}$  at  $x = 0.8c$ :  $(t/c)^{80} \geq (t/c)_{\min}^{80}$
- Prescribed minimum leading edge nose radius  
 $R_{\min}^{le}: R^{le} \geq R_{\min}^{le}$

The RAE2822 was parameterized by a volumetric NURBS. Figure 2 shows the parameterization coloured green, with the control points marked in red. The selected parameterization is a 3D control box with 2 control points in direction  $u$  (fake 3D grid), 10 in direction  $v$  and 5 in direction  $w$ . There are 14 design variables used during optimization.

Fig. 2. NURBS control box.



### 2.2. DPW wing

The DPW-W1 wing [37] [38] was selected as the initial geometry for aerodynamic optimizations. Reference quantities for this wing are given in Table 2, while Figure 3 presents the geometry

Table 2. Reference quantities for the DPW wing

$S_{ref}$ (wing reference area)	290322 mm <sup>2</sup>
$C_{ref}$ (wing reference chord)	197.55 mm
$X_{ref}$	154.24 mm (from the wing root l.e.)
$b/2$ (semi-span)	762 mm
AR (aspect ratio, $AR = b^2/S_{ref}$ )	8.0

The flow conditions and constraints of different design points were the inputs for the optimization process. These flow conditions included prescribed cruise lift, Mach number, Reynolds number as follows:

- $M = 0.76$ ,  $C_L = 0.5$ ,  $Re = 5 \cdot 10^6$  DP1 (main design point)
- $M = 0.78$ ,  $C_L = 0.5$ ,  $Re = 5 \cdot 10^6$  DP2 (high-Mach design point)

The design goal was to achieve a geometry with the minimum drag, while maintaining some specified aerodynamic and geometric constraints.

In this case, the aerodynamic constraints and penalties were:

- i. Prescribed constant lift coefficient:  
 $C_L^0 \rightarrow C_L(k) = C_L^0(k)$
- ii. Minimum pitching moment:  $C_M^0 \rightarrow C_M(k) \geq C_M^0(k)$   
 $C_L^0(k)$  and  $C_M^0(k)$  are the lift and pitching moment coefficients, respectively, of the initial geometry, for the design point  $k$ .
- iii. Drag penalty: if constraint in minimum pitching moment is not satisfied, the penalty will be 1 drag count per 0.01 in  $\Delta C_M$

while the geometric constraints were:

- i. Wing sections maximum thickness constraints:  
 $(t/c)_{section} \geq (t/c)_{section}^0$   
 where  $(t/c)_{section}^0$  is the maximum thickness for the original wing sections at the root, mid-span and tip:  
 $(t/c)_{root}^0 = (t/c)_{mid-span}^0 = (t/c)_{tip}^0 = 13.5\%$ .  
 Therefore, the maximum thickness for the optimized wing sections should be greater or equal than 13.5%.
- ii. Spar constraints: First, two locations  $(x/c)$  are defined to represent the spar positions:  
 $(x/c)_{root,1} = (x/c)_{mid-span,1} = (x/c)_{tip,1} = 0.20$   
 $(x/c)_{root,2} = (x/c)_{mid-span,2} = (x/c)_{tip,2} = 0.75$   
 Then, the thickness value of the original wing sections at these locations are defined by:  
 $(t/c)_{root,1}^0 = (t/c)_{mid-span,1}^0 = (t/c)_{tip,1}^0 = 12\%$   
 $(t/c)_{root,2}^0 = (t/c)_{mid-span,2}^0 = (t/c)_{tip,2}^0 = 5.9\%$

The parameterization defined for task 2 is depicted in Figure 4. The DPW wing was parameterized by a 3D control box with 5 control points in direction  $u$ , 10 in direction  $v$  and 5 in direction  $w$ . The parametric  $u$  direction corresponds to the  $y$  axis, the  $v$  direction to the  $x$  axis, and the  $w$  direction to the  $z$  axis. The 36 design variables to be modified are the control points in the  $w$  direction.

Fig. 3 . Planform plot (left) and 3D plot (right) of the initial DPW-W1 wing geometry

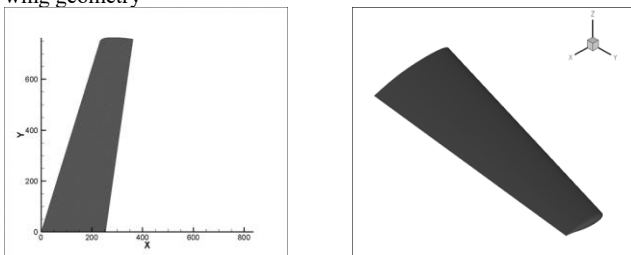
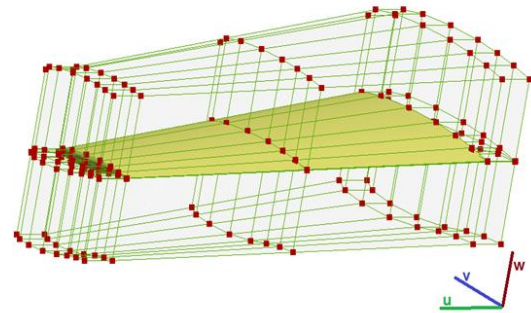


Fig. 4. DPW-W1 geometric parameterization



### 2.3. Applied approaches

The surrogate models employed by the partners are listed in Table 3.

Table 3. Summary of test cases in Task 1&2, with contributing partners and methods used

	TC1.1 RAE2822 airfoil (RANS)	TC1.2 DPW-W1 wing (Euler)	TC1.2 DPW-W1 wing (RANS)
INTA/UAH	SVMs	SVMs	SVMs
VUT	ANNs	-	-
CIRA	POD/RBF, Kriging/EGO	-	POD/RBF
FOI	Kriging, RBF	-	-
ONERA	Kriging	-	-
UNIS	Ensemble	-	-
Airbus-M	-	-	HOSVD

## 3 Optimization results

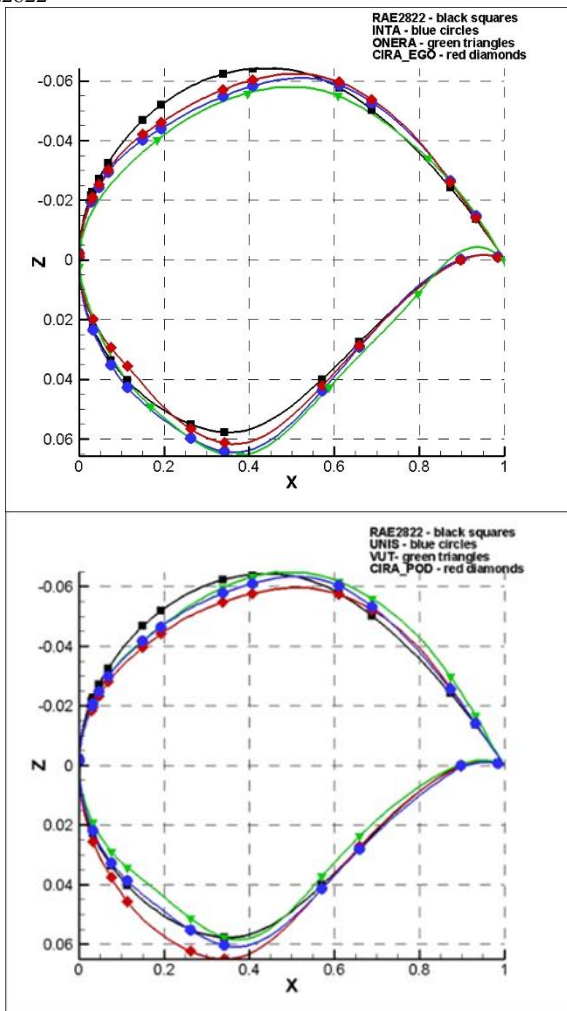
### 3.1. Task 1 RAE2822 RANS

Partner's optimized shapes are depicted in Figure 5. The objective function values obtained for each geometry after the cross validation with several solvers are summarized in Table 4.

Table 4. Average value of the Objective Function (OF) values for RAE2822 in viscous flow conditions optimization

	Mean OF (TAU, MSES, ZEN 3 levels)	Mean OF (only TAU and ZEN fine)
RAE 2822 baseline	1	1
CIRA-POD	0.6223	0.6266
CIRA-EGO	0.6208	0.6236
INTA/UAH	0.6243	0.6211
ONERA	0.6359	0.6495
UNIS	0.6367	0.6338
VUT	0.6969	0.7063

Fig. 5. Comparison of partner’s optimized geometries and baseline RAE2822



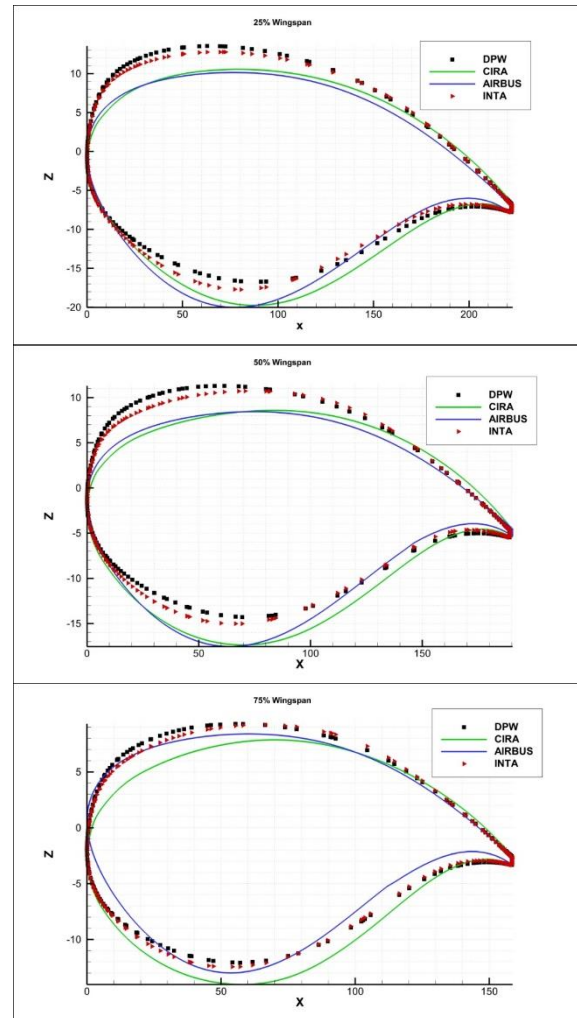
3.2. Task 2 DPW-W1 RANS

Partner’s optimized shapes at 25%, 50% and 75% wing semi-span are depicted in Figure 6. The objective function values obtained for each geometry after the cross validation with several solvers are summarized in Table 5.

Table 5. Results of cross-analysis of the optimized geometries ZEN and TAU solvers.

Geometry	Solver							OF
		CL	CD	CM	CL	CD	CM	
DPW w1	ZEN	0.5	0.0237	-0.069	0.5	0.0264	-0.078	1
DPW w1	TAU	0.5	0.0237	-0.067	0.5	0.0267	-0.070	1
CIRA	ZEN	0.5	0.0224	-0.084	0.5	0.0232	-0.089	0.91
CIRA	TAU	0.5	0.0221	-0.075	0.5	0.0233	-0.079	0.91
INTA/UAH	ZEN	0.5	0.0235	-0.084	0.5	0.0241	-0.091	0.96
INTA/UAH	TAU	0.5	0.0231	-0.074	0.5	0.0248	-0.077	0.94
AIRBUS-M	TAU	0.5	0.0231	-0.090	0.5	0.0238	-0.078	0.92

Fig. 6. Comparison of partner’s optimized geometries and baseline DPW-W1 at 25%, 50% and 75% wing semi-span.



4 Conclusions

This paper summarized the results of the GARTEUR AD/AG52 group on “Surrogate-based global optimization methods for aerodynamic design”.

Surrogate-based global optimization has been demonstrated to be feasible for aerodynamic design where there is a high number of design variables (up to 36 design variables used during optimization).

However, the accuracy of the surrogate models strongly depends on the sampling and the objective of the surrogate:

- If the objective is to provide general predictions, an a-priori LHS sampling in combination or not with Lola-Voronoi sampling seems to be a good option.
- If the objective is to better predict those regions of the design space where the optimum is located, then a mixed a-priori and adaptive sampling is recommended.

In case of optimization best results were achieved by the adaptive POD with RBF interpolation, HOSVD-based and SVMr optimization approaches.



Interested readers can consult the complete AG52 report in [www.garteur.org](http://www.garteur.org).

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