FLUTTER STABILITY ANALYSIS OF AN AIRCRAFT WING AS A FUNCTION OF THE AEROELASTIC DAMPING RATIO

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Abstract

This report provides insight into flutter stability analysis as a function of damping ratio. Flutter, an **unstable self-excited vibration** in which the structure extracts energy from the air stream and often results in catastrophic structural failure, is analyzed as dynamic instability, which may eventually result in stall or **buffeting conditions** or classical bending and torsion coupling actions. The characteristic equation of motion is analyzed in order to study stability and its relationship with the **damping ratios** on an aircraft wing. Analytical results of different mode shapes and time-dependent boundary conditions are provided in this independent research study, under the guidance of PhD. A. Vega Coso.

Those results indicate stability dependence on damping ratio as it is basically a free vibration problem. Further research is focused on considering and proving detailed results on experimental data and evaluating aeroelasticity as an essential topic on aircraft design.

Introduction

Aeroelasticity phenomenon is a combination of physical phenomena which include iteration between inertia, elastic and aerodynamic forces. It has a high impact on stability and control, -and thus, on flight mechanics-, structural vibrations and static aeroelasticity.

According to [4], **flutter** is considered one of the most important of all the aeroelastic phenomena and is the most difficult to predict. It is an unstable self-excited vibration in which the structure **extracts energy from the air stream** and often results in catastrophic structural failure [5].

This phenomena, understood as a dynamic instability, is a self-excited vibration phenomena, due to the structure own vibration motion and it does not need of any external excitation to occurs, on the contrary to what happens to forced response[1]. Furthermore, **classical bending and torsion coupling** actions occurs when the aerodynamic forces associated with motion in two modes of vibration cause the modes to couple in an unfavorable manner.

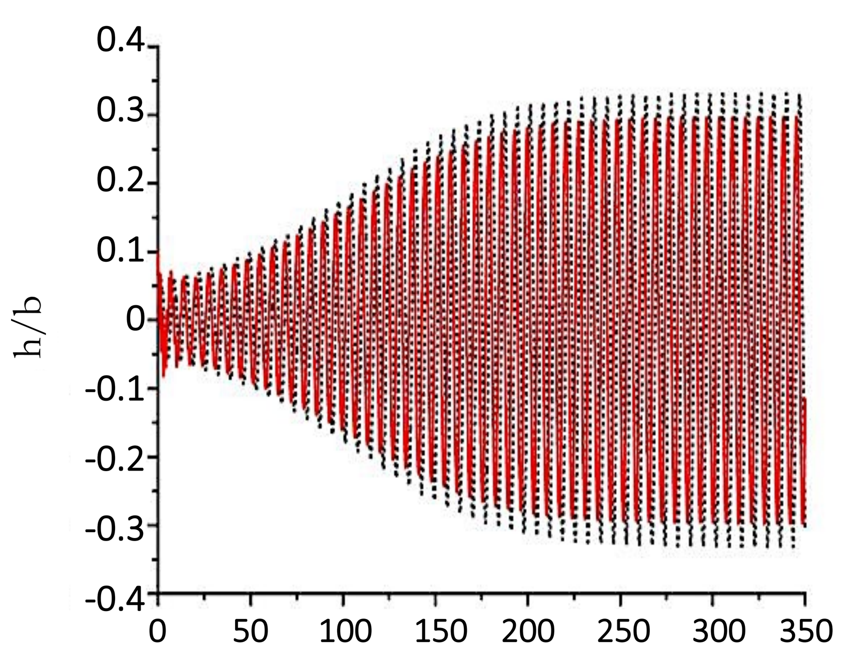

Figure 1. Limit Cycle Oscillation

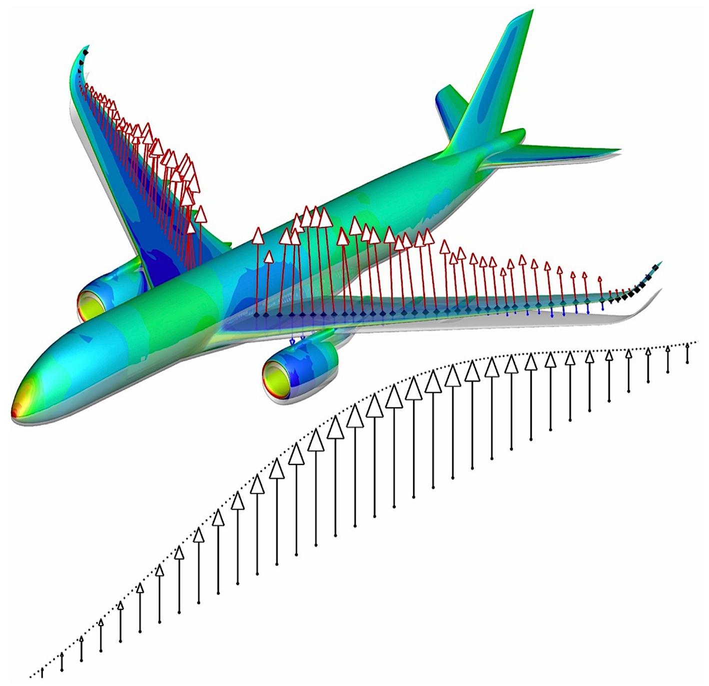

Figure 2. Gust simulation on an aircraft.

Free vibrations

Free vibrations can be defined as a system in which **no external force** is causing motion, and that the motion is primarily the result of initial conditions, such as an initial displacement of the mass element of the system from an equilibrium position and/or an initial velocity. Thus, by physically representing a **SDOF** free vibration as a damped spring-mass system, we can obtain the differential equation for damped motion of an aircraft wing:

$$m\ddot{x} + c\dot{x} + \mathrm{k}x = 0 \qquad \lambda = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

where **m** and **k** are the mass and stiffness matrices, respectively; **c**, is the damping force, and **x** is the n-dimensional column vector of generalized coordinates. The **damping ratio**(ζ) and the **undamped natural frequency**(ω_n) can be expressed:

$$\zeta = \frac{c}{2\sqrt{mk}} = \frac{c}{c_{cr}} \qquad \omega_n = \sqrt{\frac{k}{m}}$$

Results

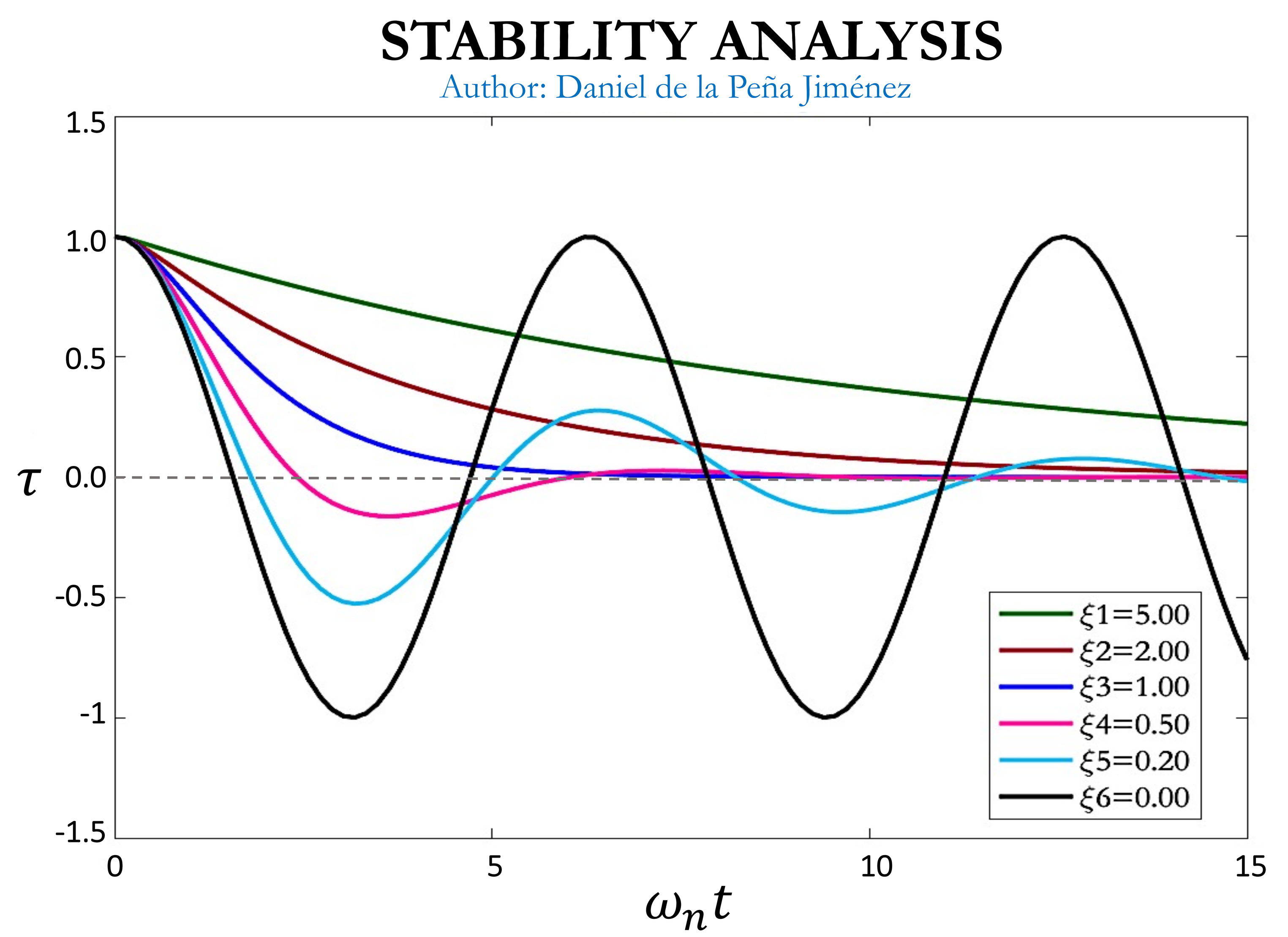

Figure 3. Stability analysis as a function of the damping ratio.

Table 1. Response classification.

Scenarios	DR	Description
OVERDAMPED RESPONSE	$\zeta > 1$	$X(t) = C_1 e^{\left(-\zeta w_n + w_n\sqrt{\zeta^2-1}\right)t} + C_2 e^{\left(-\zeta w_n - w_n\sqrt{\zeta^2-1}\right)t}$
CRITICALLY DAMPED RESPONSE	$\zeta = 1$	$X(t) = X_0 e^{(-\zeta w_n)t} + w_n X_0 t e^{(-\zeta w_n)t}$
UNDERDAMPED RESPONSE	$0 < \zeta < 1$	$X(t) = Ae^{(-\zeta w_n)t}\,[\sin(w_d t + \phi)]$
FLUTTER RESPONSE	$\zeta < 0$	Self-excitation instability

Discussion

Overdamped response results in both real roots for the characteristic equation being real and negative. This has a physical impact on the system, as the restoring force is high enough to avoid the system to oscillate. Therefore, a free force overdamped harmonic oscillator **tends to equilibrium.**

Critically damped response behaves similarly with a **faster response on the decay** with no possible oscillation. As overdamped systems, the solutions to the characteristic equations are real and negative, reaching the steady-state value the fastest without being underdamped.

Underdamped response is obtained when the damping ratio is ranged from zero to one. The response of the system is a **sinusoid function** in which energy varies while being dissipated. Reducing the damping ratio results in an oscillation in the form of $\sin(\omega_{dt} - \varphi)$, although the amplitude is constantly decreasing up to **equilibrium state** along the time.

Further discussion on **resonance** on an aircraft wing undergoing an external aerodynamic nature excitation due to atmospheric turbulence or gust is achieved by mathematically representing this phenomenon due to an **external harmonic force.**

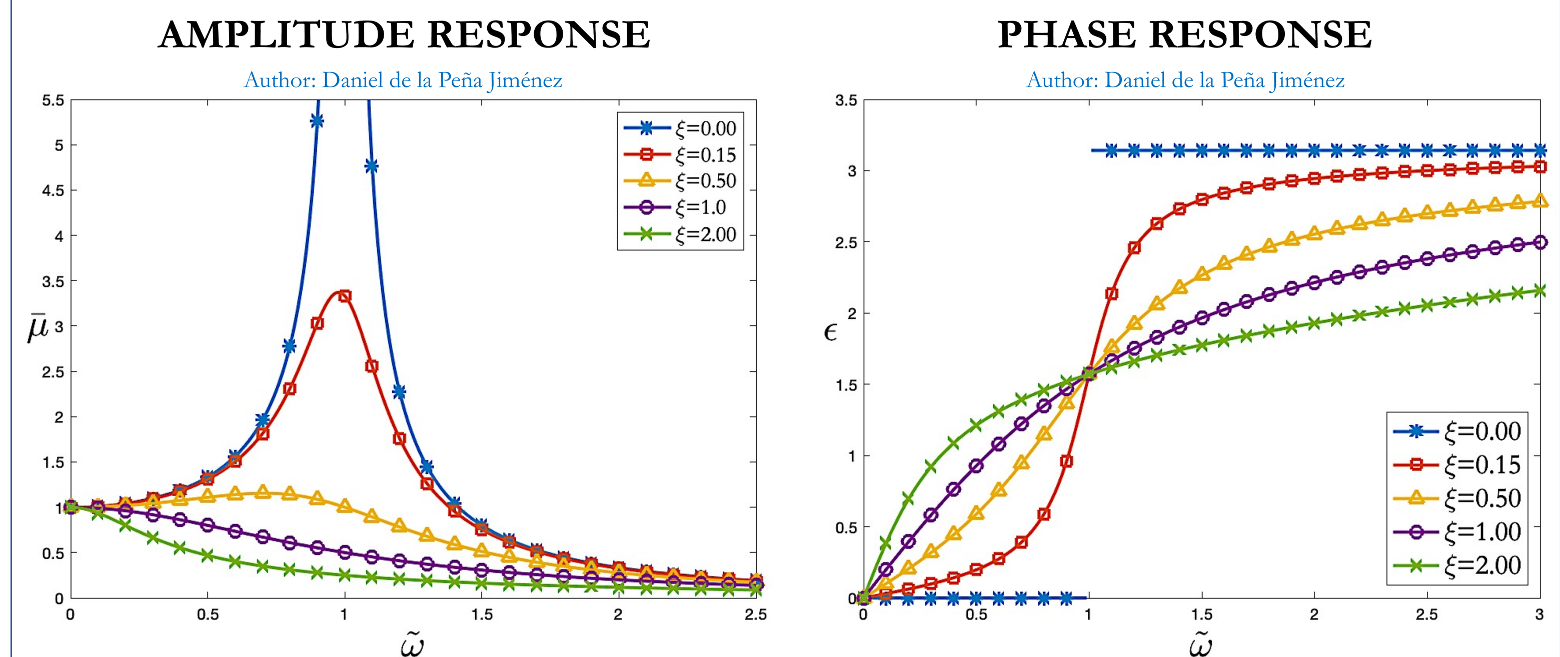

Figure 4. Normalized amplitude and phase vs normalized frequency.

It is found that the **normalized amplitude** strongly depends on the damping ratio (ζ) as the frequency for which the maximum amplitude is obtained shifts away from the **natural frequency** when the damping ratio increases. Also, this excitation frequency increases as **phase** increases from 0 to π. At $\pi/2$, the natural frequency is equal to the **excitation frequency** of the gust.

Conclusions

Stability dependence on damping ratio is achieved by integrating a free vibration problem and the equations of motion. The damping ratio coefficient can be modified by applying the following design solutions:

- Adjust the stiffness of the aircraft wing structure.
- Change geometrical wing parameters.
- Distribute mass for damping ratio stability.

References:

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